INVESTIGATING THE QCD SCALE DEPENDENCE OF TOTAL CROSS SECTION FOR HEAVY QUARK PRODUCTION IN $p\bar{p}$ COLLISIONS

A. MIRJALILI*,†, A. N. KHORRAMIAN*† and S. ATASHBAR TEHRANI*‡

†Physics Department, Yazd University, Yazd, Iran
‡Physics Department, Semnan University, Semnan, Iran
§Physics Department, Persian Gulf University 75168, Boushehr, Iran

*Institute for Studies in Theoretical Physics and Mathematics (IPM), School of Physics
P. O. Box 19395-5531, Tehran, Iran

E-mails: khorramiana@theory.ipm.ac.ir, mirjalili, atashbar@ipm.ir

There is a sizable and systematic discrepancy between experimental data on the $b\bar{b}$ production in $pp$, $\gamma p$ and $\gamma\gamma$ collisions and the existing theoretical calculations within perturbative QCD. The uncertainty is coming from renormalization and factorization scale dependence of finite order perturbative calculations of the total cross section of $b\bar{b}$ production in such collisions and will be discussed for $pp$ collision in detail. If we employ the approach of "Complete RG-improvement (CORGI)"*, in which one should perform a resummation to all-orders of renormalization and factorization group-predictable terms at each order of perturbation theory, then the scales dependence will be avoided and the mentioned discrepancy is reduced significantly.

1 Introduction

One of the clean test of perturbative QCD is heavy quark production in hard collisions of hadrons, leptons and photons. Recent data on $b\bar{b}$ production in $p\bar{p}$ collisions at the Tevatron, $\gamma p$ collision in HERA and $\gamma\gamma$ collisions in LEP$^2$ indicates that they lie systematically by a factor of about 2 – 4 above the median of current theoretical calculations. The QCD calculations for the above cases depend on a number of inputs, for instance, $\alpha_s$, parton distribution functions (PDF) of colliding hadrons or photons, ..., and finally, the choice of renormalization (RS) and factorization (FS) scales $\mu$ and $M$. We are going to investigate the dependence of existing fixed order (LO and NLO) QCD calculation of the total cross section in $p\bar{p}$ collisions on the choice of the renormalization and factorization scales. Further, we indicate whether these scales will be avoided from our calculation, if we do a resummation on all ultraviolet terms which involve these two scales. Finally, we expect to have a more consistent theoretical result comparing to the available experimental data.

2 Basic Definitions

The basic quantity of perturbative QCD calculations, the renormalized color coupling $\alpha_s(\mu)$, depends on the renormalization scale $\mu$ which is governed by

$$\frac{\partial \alpha_s(\mu)}{\partial \ln \mu^2} = \beta(\alpha_s(\mu)) = -\frac{\beta_0}{4\pi} \alpha_s(\mu)^2 - \frac{\beta_1}{16\pi^2} \alpha_s(\mu)^3 + \cdots,$$

where for $n_f$ massless quarks $\beta_0 = 11 - \frac{2}{3}n_f$ and $\beta_1 = 102 - \frac{38}{3}n_f$.

The solutions of the above equation depend, beside $\mu$, on renormalization scheme (RS). At NLO calculation this RS can be specified, for instance, via the parameter $\Lambda_{RS}$, corresponding to the renormalization scale for which $\alpha_s(\mu)$ diverges.

At two loop calculations the coupling $\mu$ is then given as the solution of the equation

$$\frac{\beta_0}{4\pi} \ln \left( \frac{\mu^2}{\Lambda_{RS}^2} \right) = \frac{1}{\alpha_s(\mu)} + c \ln \frac{\alpha_s(\mu)}{1 + \alpha_s(\mu)},$$

where $c = \frac{\beta_1}{4\pi\beta_0}$. For hadrons, the factorization scale dependence of PDF (M) is determined by the system of evolution equations for quark singlet, non-singlet and gluon distribution functions.

$$\frac{d\Sigma(M)}{d\ln M^2} = P_{qq}(M) \otimes \Sigma(M) + P_{qG}(M) \otimes G(M),$$

$$\frac{dG(M)}{d\ln M^2} = P_{qG}(M) \otimes \Sigma(M) + P_{GG}(M) \otimes G(M),$$

*Talk presented by A.M. at the XIth Regional Conference on Mathematical Physics, May 2004, Tehran, Iran.
\[
\frac{dq_{NS}(M)}{d\ln M^2} = P_{NS}(M) \otimes q_{NS}(M) + P_G(M) \otimes G(M),
\]
where
\[
\Sigma(x, M) = \sum_{i=1}^{N_f} (q_i(x, M) + \bar{q}_i(x, M)),
\]
\[
q_{NS,i}(x, M) = (q_i(x, M) + \bar{q}_i(x, M)) - \frac{1}{n_f} \Sigma(x, M),
\]
for all \(i\). The cross symbol in above relations indicates the convolution
\[
f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y). \tag{7}
\]
The splitting functions admit expansion in powers of \(\alpha_s(M)\)
\[
P_{ij}(x, M) = \frac{\alpha_s(M)}{2\pi} P_{ij}^{(0)}(x) + \left(\frac{\alpha_s(M)}{2\pi}\right)^2 P_{ij}^{(1)}(x)
+ \cdots, \tag{8}
\]
where \(P_{ij}^{(0)}(0)\) are unique and as all higher splitting functions \(P_{ij}^{(j)}, j > 0\) depend on the choice of the factorization scheme (FS). Conversely, they can be taken to define the FS.

### 3 RS and FS Dependence of Coefficients

For a single case of a dimensional observable \(R(Q)\) with
\[
R(Q) = a + r_1 a^2 + r_2 a^3 + \ldots + r_n a^{n+1} + \cdots, \tag{9}
\]
the RS can be labeled by the non-universal coefficients of the \(\beta\)-function and \(\Lambda_{RS}\).\(^6\)

Self-consistency of perturbation theory that is the derivative of \(N\)-th order approximant \(R(n)\) with respect to \(\ln(\mu)\) is of higher order than the approximant itself. It will yield
\[
r_1(Q, \mu, RS) = k b \ln\left(\frac{\mu}{Q}\right) + r_1(1, RS),
= k b \ln\left(\frac{\mu}{\Lambda_{RS}}\right) - \rho(Q), \tag{10}
\]
where
\[
\rho(Q) = k b \ln\left(\frac{Q}{\Lambda_{RS}}\right) - r_1(1, RS), \tag{11}
\]
is a renormalization scale and scheme invariant. For other coefficients, we will arrive first on following partial derivatives for \(r_2, r_3, \ldots\)
\[
\frac{\partial r_2}{\partial r_1} = 2r_1 + c, \quad \frac{\partial r_2}{\partial c_2} = -1, \quad \frac{\partial r_2}{\partial c_3} = 0, \ldots,
\]
\[
\frac{\partial r_3}{\partial r_1} = c_2 + 2c r_1 + 3r_2, \quad \frac{\partial r_3}{\partial c_2} = -2r_1, \quad \frac{\partial r_3}{\partial c_3} = -1, \ldots
\]
After integration, one finds
\[
r_2(r_1, c_2) = r_1^2 + c r_1 + X_2 - c_2, \tag{12}

r_3(r_1, c_2, c_3) = r_1^3 + \frac{5}{2} c r_1^2 
+ (3X_2 - 2c_2)r_1 + X_3 - \frac{1}{2} c_3, \tag{13}
\]
The general structure is as follows
\[
r_n(r_1, c_2, \ldots, c_n) = r_n(r_1, c_2, \cdots, c_{n-1}) 
+ X_n - c_n/(n - 1). \tag{13}
\]
Where \(X_n\) are \(Q\)-independent and RS-invariant and are unknown unless a complete \(N^nLO\) calculation has been performed.

### 4 Complete RG Improvement

From pervious results for \(r_1, r_2, \ldots\), we can reformulated \(R(Q)\) as
\[
\mathcal{R}(Q) = a + r_1 a^2 + (r_1^2 + c r_1 + X_2 - c_2)a^3 + \cdots. \tag{14}
\]
Given a NLO calculation, \(r_1\) is known but \(X_2, X_3, \ldots\) are unknown. Thus, the complete subset of known terms in (13) at NLO is
\[
a_0 = a + r_1 a^2 + (r_1^2 + c r_1 - c_2)a^3 + \cdots, \tag{15}
\]
and it is RS-invariant. Choose \(r_1 = 0, c_2 = c_3 = \cdots = c_n = 0\), we obtain \(a_0 = a\). At NNLO calculation \(X_2\) is unknown. Further, infinite subset of terms are known and can be resummed to all orders\(^4\)
\[
X_2 a_0^3 = X_2 a^3 + 3X_2 r_1 a^4 + \cdots. \tag{16}
\]
Finally, we will arrive at
\[
\mathcal{R}(Q) = a_0 + X_2 a_0^3 + X_3 a_0^4 + \cdots + X_n a_0^{n+1} + \cdots, \tag{17}
\]
where \(a_0 = a(0, 0, 0, \cdots)\) is the coupling in this scheme and satisfies
\[
\frac{1}{a_0} + c n\left(\frac{c_0 a_0}{1 + c_0 a_0}\right) = b \ln\left(\frac{Q}{\Lambda'_{RS}}\right). \tag{18}
\]
In fact, the solution of this transcendental equation can be written in closed form in terms of the Lambert $W$-function, defined implicitly by $W(z) \exp(W(z)) = z$,

$$a_0 = -\frac{1}{c[1 + W(z(Q))]} ,$$

$$z(Q) = -\frac{1}{e} \left( \frac{Q}{\Lambda_R} \right)^{-b/c} .$$

(19)

5 Other Approaches in Investigating Total Cross Section of Heavy Pair Production

In addition to the previous approach, to deal perturbative QCD for what we concern, we can have the following approaches:

**Principle of Minimal Sensitivity (PMS):** Here, the emphasize is put on the stability of the results with respect to variable $\mu$. In absence of information on higher perturbative terms, the PMS approach is natural as it selects the point $\mu$ where the truncated perturbative expansion is most stable and has thus locally the property possessed by all the other result globally.

**The effective charge (EC):** It is based on the criterion of apparent convergence of perturbation expansions and the renormalization scale $\mu$ is chosen in such a way that all higher order contributions vanish i.e. demanding $R(n) = R(k)$ for all $k$.

**Complete RG Improvement (CORG):** This approach is explained in more details in the previous section. Due to the properties of this approach where we encounter with RS invariant quantities, we expect to have more consistent result comparing to experimental data, specially for total cross section of heavy quark pair production.

6 Conclusions

Here, the dependence on the factorization scale for total cross section is canceled by that of PDF, provided the splitting function $P_{ij}$ in the evolution equation is taken to all orders. At NLO, i.e. taking into account the first two terms in related perturbative series, we get

$$\sigma_{tot}^{NLO}(M, \mu) = \alpha_s^2(\mu) \int \int dxdy \sum_{i=1}^{2n_f} q_i(x, M)$$

$$\times q_i(y, M)[\alpha_s^{(2)}(xy) + \alpha_s^{(3)}(xy, M, \mu)]$$

$$+ 2 \alpha_s^2(\mu) \int dx dy \Sigma(x, M) G(y, M) \alpha_s(\mu) \sigma_{GG}^{(2)}(xy, M)$$

$$+ \alpha_s^2(\mu) \int dx dy G(x, M) G(y, M) [\sigma_{GG}^{(2)}(xy) + \alpha_s(\mu)$$

$$\times \sigma_{GG}^{(3)}(xy, M, \mu)].$$

Here, the sum is over $n_f$ quarks and antiquarks and the relation between PDF of protons and antiprotons was taken into account. At this stage, we can employ the CORGI approach, where we need to use it first in moment space to extract parton distributions and then, we have to reformulate in this approach the partonic cross section. Consequently, we expect to get better consistency between our theoretical model and experimental data for cross section of $b \bar{b}$ production, which will be a new task in our future attempts.

Acknowledgements

We would like to thank the “Institute for studies in theoretical Physics and Mathematics (IPM)” to perform such a rich conference and to support and give us the opportunity to present our research works.

References