QCD analysis of polarized structure functions in next-to-leading-order, using improved valon model

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Abstract. Polarized parton distribution functions (PPDF) has been calculated in the improved valon model using next-to-leading order approximation. The Bernstein polynomial method has been applied to do direct fits. Our predictions for next-to-leading order of polarized parton distributions and proton structure function have been compared with leading order.

INTRODUCTION

Next-to-Leading Order(NLO) for unpolarized parton distributions in framework of valon model and a leading-order(LO) QCD analysis of polarized case has been performed as well in this frame [1, 2]. The valon model to polarized case in NLO approximation in QCD are employed. Here the distribution of unpolarized valons were calculated firstly by Hwa [3] which is a successful model in describing the internal hadronic structure. Afterward a new set of parameters which describe unpolarized valon distributions in proton were determined [4]. It is shown that polarized parton distribution functions are in connection with unpolarized ones [5]. According to this view we supposed the same relations between U and D polarized valon distribution functions and unpolarized ones. In this connection some unknown parameters are appeared which can be determined by using the Bernstein averages fitting method [6].

MOMENTS ANALYSIS OF POLARIZED VALON AND RELATED PARTON DISTRIBUTIONS IN NLO

In Ref.[4] the unpolarized valon distribution have been introduced and calculated in the following form:

\[ G_U/p(y) = 72.49y^{1.75}(1-y)^{3.8}, \quad G_D/p(y) = 38.69y^{1.05}(1-y)^{4.51}, \]

(1)

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where \( G_{U,D/p}(y) \) are the probability of finding unpolarized \( U, D \)-valon with momentum fraction \( y \) in unpolarized hadron. According to [2] which is corresponded to LO calculations, we can define as well in the NLO the polarized valon distributions from the unpolarized valon ones in the following form

\[
\delta G_{j/p}^{NS}(y) = \delta F_{j}^{NS}(y) \times G_{j/p}(y),
\]

\[
\delta G_{j/p}^{S}(y) = \delta F_{j}^{S}(y) \times G_{j/p}(y),
\]

where the subscript \( j \) refer to \( U \) and \( D \) valon type, and \( G_{j/p}(y) \) in Eq. (2) is the probability of finding a \( j \)-valon with momentum fraction \( y \) in a polarized proton. In Eq. (2) we assumed the functional form of \( \delta F_{j}^{NS}(y) \) and \( \delta F_{j}^{S}(y) \) as follows

\[
\delta F_{j}^{NS}(y) = N_{j} \gamma_{j}(1 - y)^{B_{j}} (1 + \gamma_{j}y + \eta_{j}y^{0.5}),
\]

\[
\delta F_{j}^{S}(y) = \delta F_{j}^{NS}(y) \times (\kappa y^{0.5} + \lambda y + \mu y^{1.5} + \nu y^{2.5} + \tau y^{3}).
\]

Let us define the Mellin moments of polarized valon distribution functions, \( \delta G_{j/p}^{NS, S} \), as in below:

\[
\delta M_{j/p}^{NS, S}(n) \equiv \int_{0}^{1} y^{n-1} \delta G_{j/p}^{NS, S}(y) dy.
\]

By inserting the Eq.(2) in above equation, the final results as a function of \( n \) which involves the unknown parameters of Eqs. (3,4) will be appeared. To calculate the NLO evolutions of the polarized parton distributions in the valon, we used its moments. The non-singlet(NS) part evolves according to

\[
\Delta M_{NS} = \left( 1 - \frac{\alpha_{s}(Q^{2}) - \alpha_{s}(Q_{0}^{2})}{2\pi} (\delta d_{NS}^{(1)n} + \frac{2\pi b'}{b} \delta d_{qq}^{(0)n}) \right) L \delta d_{\tilde{q}q}^{(0)n},
\]

where \( L(Q^{2}) \equiv \frac{\alpha_{s}(Q^{2})}{\alpha_{s}(Q_{0}^{2})} \). The evolution in the flavor singlet and gluon sector are governed by \( 2 \times 2 \) the anomalous dimension matrix with the explicit solution given by

\[
\left( \begin{array}{c}
\delta M_{S} \\
\delta M_{\tilde{q}q}
\end{array} \right) = \left( L \delta \tilde{d}_{\tilde{q}q}^{(0)n} + \frac{\alpha_{s}(Q^{2})}{2\pi} \hat{U} \delta \tilde{d}_{\tilde{q}q}^{(0)n} - \frac{\alpha_{s}(Q_{0}^{2})}{2\pi} L \delta \tilde{d}_{\tilde{q}q}^{(0)n} \hat{U} \right) \left( \begin{array}{c}
1 \\
1
\end{array} \right),
\]

where \( \delta M_{\tilde{q}q} \) is the spin dependent quark-to-gluon evolution function. All associated functions and quantities in above equations have been defined in Ref.[5].

**NLO MOMENTS OF PROTON PPDF’S AND STRUCTURE FUNCTION**

Having obtained the moments of polarized valon distributions, the determination of the moments of parton distributions in a proton are straightforward. Distributions that we shall calculate are \( \delta u_{v}, \delta d_{v}, \delta \Sigma \) and \( \delta g \). Their moments are denoted respectively by:"
\( \delta u^n_v(Q^2), \delta d^n_v(Q^2), \delta \Sigma^n(Q^2) \) and \( \delta g^n(Q^2) \). Therefore the moments of polarized \( u \) and \( d \)-valence quark in a proton can be indicated by:

\[
\delta u^n_v(Q^2) = 2\delta M^n_{U/p}(n) \times \delta M_{NS+}(n, Q^2), \quad (8)
\]
\[
\delta d^n_v(Q^2) = \delta M^n_{D/p}(n) \times \delta M_{NS+}(n, Q^2), \quad (9)
\]

the factor 2 in Eq.(8) backs to existence of 2-\( U \) type valons. The moment of polarized singlet distribution (\( \Sigma \)) and gluon distribution are as follows:

\[
\delta \Sigma^n(Q^2) = (2\delta M^n_{U/p} + \delta M^n_{D/p}) \times \delta M_S(n, Q^2), \quad (10)
\]
\[
\delta g^n(Q^2) = (2\delta M^n_{NS+} + \delta M^n_{NS}) \times \delta M_{gq}(n, Q^2). \quad (11)
\]

In Eq.(10) \( \Sigma \) symbol indicates \( \Sigma_q = u, d, s(q + \bar{q}) \), thus by having \( \Sigma \) contribution, and all valence quark in moment space, the contribution of \( \delta \bar{q} \) can be specified directly. Also in Eq. (11), \( \delta M_{gq}(n, Q^2) \) is the quark-to-gluon evolution function. As we know in the NLO contributions to \( g_1(x, Q^2) \) we can use directly its moment in following form

\[
g^n_1(Q^2) = \frac{1}{2} \sum_q e^2_q (1 + \frac{\alpha_s}{2\pi} \delta C^n_q [\delta q^n(Q^2) + \delta \bar{q}^n(Q^2)] + \frac{\alpha_s}{2\pi} 2\delta C^n_g \delta g^n(Q^2)), \quad (12)
\]

here \( \delta q^n(Q^2), \delta \bar{q}^n(Q^2) \) and \( \delta g^n(Q^2) \) are moments of polarized parton distributions in a proton. Also \( \delta C^n_q, \delta C^n_g \) are the \( n \)-th moment of spin-dependent Wilson coefficients given by Ref.[5]. Since now the moments of polarized parton distributions has been determined, we can obtain the moment of polarized proton structure function in NLO by inserting the required distributions function in Eq. (8-11). According to Eqs. (3,4), which involves 16 unknown parameters and that they inter in sequence relations (8-11) and finally Eq. (12), it is obvious that the final version for \( g^n_1(Q^2) \), includes as well these unknown parameters. If we can calculate the unknown parameters then the computation of all moments of polarized parton distributions and structure function, \( g^n_1(Q^2) \), are possible.

**QCD FITS TO EXTRACT POLARIZED VALON DISTRIBUTIONS IN NLO**

Because for a given value of \( Q^2 \), only a limited number of experimental points, covering a partial range of values \( x \), are available, one can not use the moments directly. A method devise to deal to this situation is that to take averages of structure functions with Bernstein polynomials [6]

\[
g^n_{n,k} = \frac{(n-k)!\Gamma(n+2)}{\Gamma(k+1)\Gamma(n-k+1)} \sum_{l=0}^{n-k} \frac{(-1)^l}{l!(n-k-l)!} \bar{g}^{(k+l)+1}_1(Q^2), \quad (13)
\]

where

\[
\bar{g}^{(k+l)+1}_1(Q^2) = \int_0^1 x^{(k+l)+1} g_1(x, Q^2)dx. \quad (14)
\]
The Eq. (13) represent averages of function \( g_1(x, Q^2) \) in the region \([\bar{x}_{n,k} - \frac{1}{2}\Delta x_{n,k}, \bar{x}_{n,k} + \frac{1}{2}\Delta x_{n,k}]\) [7]. To obtain experimental averages \( g_{n,k} \) from the E143 and SMC data for \( xg_1 \) [8], we fit \( xg_1(x, Q^2) \) for each bin in \( Q^2 \) separately, to the convenient phenomenological expression

\[
xg_1^{(\text{phen})} = \mathcal{A} x^p (1-x)^q.
\]

Using Eqs. (13,14) the Bernstein averages \( g_{n,k}(Q^2) \) can be written in terms of odd and even moments

\[
g_{2,1}(Q^2) = 6 \left( g_1^1(Q^2) - g_1^3(Q^2) \right),
\]

\[
g_{2,2}(Q^2) = 3 \left( g_1^3(Q^2) \right),
\]

\[
\vdots
\]

We shall use the result of Eq.(12) for the QCD prediction of \( g_1^i(Q^2) \). The basic unknown fit parameters will be, \( N_U, \alpha_U, \beta_U, \gamma_U, \ldots, \rho, \tau \). Thus there are 16 parameters to be simultaneously fitted to the experimental \( g_{n,k}(Q^2) \) averages. After obtain 16 unknown parameters according to Eqs. (3,4), we are in situation to introduce polarized valon distributions which defined before in Eq. (2). In Fig.(1) we plotted \( 2 y \delta G_{NSU/p}^N + y \delta G_{NSD/p}^N \) and \( 2 y \delta G_{SU/p}^N + y \delta G_{SD/p}^S \) as a function of \( y \) in LO and NLO approximations.

**FIGURE 1.** The plots of \( 2 y \delta G_{NSU/p}^N + y \delta G_{NSD/p}^N \) and \( 2 y \delta G_{SU/p}^N + y \delta G_{SD/p}^S \) as a function of \( y \).

**X-SPACE PPDF’S AND POLARIZED PROTON STRUCTURE FUNCTION**

By using convolution integral as following

\[
\delta q_{i/p}(x, Q^2) = \sum_j \int_x^1 \frac{dy}{y} \delta G_{j/p}(y) \delta f_{i/j}(x, Q^2),
\]

(16)
where the summation is over different types of valons, we can obtain the PPDF’s in the proton in $x$-space. In this equation we need in addition to $\delta G_{j/p}(y)$, to know the corresponding polarized $i$-parton distributions in a $j$-valon, $\delta f_{i/j}(z = \frac{x}{y}, Q^2)$. To obtain the $z$-dependence of parton distributions we can employ the inverse Mellin transform technique [5]. Now we can calculate all polarized parton distributions in a proton as a function of $x, Q^2$ by using convolution integral. We presented in Fig.(2) the results of $xg_1^p(x, Q^2)$, for $Q^2 = 2, 5 \text{ (GeV}^2\text{)}$ and compared it with experimental data Ref.[8].

![FIGURE 2. Polarized proton structure function](image)

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